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Enforcing Force-Displacement Proportionality in Nonlinear Systems through the Addition of Nonlinearity

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Abstract

Linear systems obey the principle of superposition, i.e., they are characterized by a proportionality between their response and the applied force. Because of the scaling that exists between different forcing levels, engineering design is therefore greatly facilitated. However, nonlinearity is a frequent occurrence in engineering structures and is such that the superposition principle can generally no longer be applied.

The objective of the present study is to introduce nonlinearity into an already nonlinear system to retrieve certain dynamical properties exhibited by linear systems, hence creating a sort of compensation effect. For instance, reference [1] enforced isochronicity, i.e., amplitude-independent resonant frequency, in nonlinear systems. The focus herein is to investigate how the proportionality between the response of the system and the applied force can be retrieved for a specific vibration mode in a large range of forcing levels.

Considering a general mechanical system, described by the system of differential equations

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} + \mathbf{b}_{nl} = f_0 \cos(\omega t)\mathbf{f}, \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the mass, damping and stiffness matrices, respectively, $\mathbf{b}_{nl}$ includes the nonlinear terms, f_0 is the forcing amplitude, ω is the excitation frequency, t is time and $\mathbf{f}$ is a constant vector that locates the applied force. For simplicity, we consider that the system has a single nonlinear element described by a third-order term, $\mathbf{b}_{nl} = \tilde{\alpha}_3 \tilde{\mathbf{b}}_{nl}$, where $\tilde{\alpha}_3$ is a real parameter.

Normalizing the system using $\mathbf{y} = \mathbf{x}/f_0$, the forcing amplitude and the nonlinearity are expressed by the unique parameter $\alpha_3 = \tilde{\alpha}_3 f_0^2$. Applying the harmonic balance method, Eq. (1) can be expressed as a system of nonlinear algebraic equations, $\mathbf{A}(\omega)\mathbf{q} + \alpha_3 \mathbf{d}(\mathbf{q}) = \mathbf{c}(\omega)$, where $\mathbf{q}$ collects the amplitude of the different harmonics of the solution, $\mathbf{d}$ contains the nonlinear terms and $\mathbf{c}$ is related to the external forcing.

We introduce in the system another third-order nonlinearity, $\beta_3 = b_3\alpha_3$. Thus, the system of algebraic equations becomes $\mathbf{A}\mathbf{q} + \alpha_3\left(\mathbf{d} + b_3\mathbf{d}_\beta\right) = \mathbf{c}$. Expanding $\mathbf{q}$ with respect to α_3 as $\mathbf{q} = \mathbf{q}_0 + \alpha_3\mathbf{q}_1 + O(\alpha_3^2)$, the approximated frequency response of the system can be obtained explicitly, i.e., $\mathbf{h} = \mathbf{h}_0 + \alpha_3\left(\mathbf{h}_{10} + b_3\mathbf{h}_{13}\right) + O(\alpha_3^2)$, where $\mathbf{h}_0$, $\mathbf{h}_{10}$ and $\mathbf{h}_{13}$ depend on ω. Focusing on the frequency response of the first coordinate, we call $h = \mathbf{h}(1)$ and analogously h_0, h_{10} and h_{13} for $\mathbf{h}_0$, $\mathbf{h}_{10}$ and $\mathbf{h}_{13}$.

We call $\bar{\omega}_0$ the resonant frequency of the considered mode of the underlying linear system, and ω_0 the corresponding resonant frequency when the two nonlinearities are considered. Approximating the difference between ω_0 and $\bar{\omega}_0$ with a linear function proportional to α_3, the value of b_3 that satisfies, in first approximation, the proportionality relation expressed through the objective function

$H = -h_0(\bar{\omega}_0) + h(\omega_0) = 0$ is

$$b_3 = -\frac{h_{0\omega}h_{10\omega} - h_{10}h_{0\omega\omega}}{h_{0\omega}h_{13\omega} - h_{13}h_{0\omega\omega}}\Big|_{\omega=\bar{\omega}_0}, \tag{2}$$

where the subscript $_\omega$ indicates derivation with respect to ω.

The developments were validated using a two-degree-of-freedom (2DOF) system possessing a cubic spring:

$$\begin{aligned} m_1\ddot{x}_1 + k_1x_1 + c_2(\dot{x}_1 - \dot{x}_2) + k_2(x_1 - x_2) + \tilde{\alpha}_3x_1^3 &= f_0\cos\omega t, \\ m_2\ddot{x}_2 + c_2(\dot{x}_2 - \dot{x}_1) + k_2(x_2 - x_1) &= 0. \end{aligned} \tag{3}$$

Fig. 1(a) illustrates that the original nonlinear system is characterized by a strong dependence of the resonance peak amplitude to forcing level. An additional cubic spring with coefficient given by (2) was then introduced in the system between masses m_1 and m_2. Fig. 1(b) shows that the force-displacement proportionality can be (almost) maintained in a certain range of forcing amplitudes.

In order to extend the range of displacement-force proportionality, the procedure was generalized to nonlinearities possessing additional higher-order terms. For instance, Fig. 1(c) depicts the improvement that can be obtained when a quintic spring is added in parallel to the cubic spring. A resonance peak that exhibits no visible dependence on forcing amplitude is obtained for variations of the resonant frequency up to 20 %; this linear-like regime is then followed by a sudden detuning.

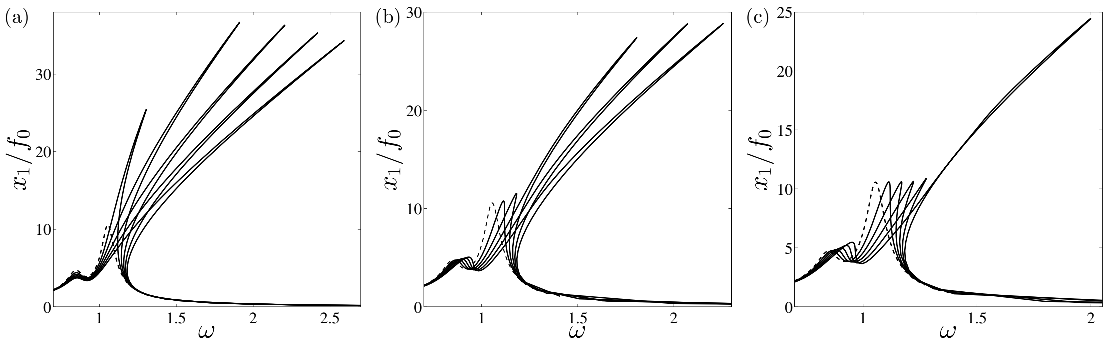

Figure 1: Normalized frequency response of a 2DOF system for different values of the forcing amplitude. (a) Original nonlinear system; (b) system with an added cubic nonlinearity; (c) system with added cubic and quintic nonlinearities. Dashed line: underlying linear system.

Devices that work properly for linear systems, but fail if the system is nonlinear, represent a possible application of this procedure. This is for instance the case of the nonlinear tuned vibration absorber proposed in [2] that can mitigate a nonlinear resonance in a large range of forcing amplitudes.

References

[1] Kovacic, I.; Rand, R.: About a class of nonlinear oscillators with amplitude-independent frequency. Nonlinear Dynamics, Vol. 74, No. 1-2, pp. 455–465, 2013.

[2] Habib, G.; Detroux, T.; Viguié, R.; Kerschen, G.: Nonlinear generalization of Den Hartog's equal peak method. Mechanical Systems and Signal Processing, Vol. 52-53, pp. 17–28, 2015.